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NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 199

Application of an Iterative Variational Method with Balanced Constraint to Hemispheric Analysis and Prediction

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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.

1. Introduction

An iterative method of numerical variational analysis for adjusting wind and height fields at constant pressure surfaces has been under development at NMC. This variational method uses the general balance equation as the dynamic constraint, and principles of variational calculus to derive a system of "Euler-Lagrange" equations. The "Euler-Lagrange" equation are solved by the method of relaxation with proper boundary conditions. Solutions to the system of equations determine the magnitude of modification to the existing fields of wind and height on isobaric surfaces. The formulation of the variational method was described in detail in a previous NMC Office Note (Gerrity and Chu, 1978).

In this paper, the application of this variational method to an actual case of hemispheric analyses and the resulting modifications to the analyses are reported. The modified analyses of isobaric wind and height fields are then used to replace the operationally analyzed fields, and to generate a 7-layer PE (Primitive Equation) model forecast up to 48 hours. The forecast fields made with the modified analyses are later compared with those generated with the operational analyses without modification.

2. Applying the Variational Method to Hemispheric Analyses

The variational method is designed to be applied to an existing meteorological field. In other words, it operates to modify meteorological variables that are already on grid points. It is usually used to modify an analyzed field, although the application needs not be limited to only analyzed values of a meteorological variable on grid points. In this paper, the variational method is applied to the Hough analyses, which are the operational hemispheric analyses on the 65 x 65 polar stereographic grid at NMC.

In the previous NMC Office Note (Gerrity and Chu, 1978) it was shown that the "Euler-Lagrange" equations can be combined to yield the following

$$\nabla^2\lambda + N^* = 0 \tag{1}$$

where λ is the Lagrange multiplier, N^* is a measure of imbalance between the existing wind and height fields as prescribed by the general balance equation. N^* can be computed from the analyzed fields of wind and height in the following manner:

$$N^* = \nabla^2\phi - f_o\nabla^2\psi - \nabla\cdot(\eta - f_o)\nabla\psi + \tfrac{1}{2}\nabla^2(\nabla\psi\cdot\nabla\psi) \tag{2}$$

where ϕ is the geopotential, ψ the stream function, η the absolute vorticity, and f_o is a constant (10^{-4} sec^{-1}). The last two terms are the ageostrophic terms.

Equation (1) is solved by the relaxation method using the homogenous Dirichlet boundary condition on λ. Once λ is known, the changes in ϕ and ψ can be calculated from the following additional boundary conditions:

$$\phi_c = \tfrac{1}{2}\lambda \tag{3}$$

$$\psi_c = -\frac{1}{2f_o}\lambda \tag{4}$$

where ϕ_c is a change in ϕ, and ψ_c in ψ.

The iteration process proceeds in the following way. (1) calculate N^* from the wind (through ψ) and height (through ϕ) fields from equation

(2), (2) solve Equation (1) for λ with calculated N^*, (3) compute height change through ϕ_c from Equation (3), and (4) compute changes in u and v components of the wind through ψ_c from Equation (4). The whole process is iterated until the height change through ϕ_c at all interior grid points is equal to or smaller than a prespecified convergence criterion. This convergence criterion is 1 meter at all pressure levels from 1000 mb to 250 mb. At pressure levels higher than 250 mb, a less stringent convergence criterion of 10 meters ensures convergence and solutions to Equation (1).

3. A Case of Hemispheric (Hough) Analyses and Their Modification

An archived case of 12Z, 9 January 1975, was used in our experiment of applying variational method with the general balance equation as the dynamic constraint. This case is well known at NMC and was used by Phillips (1978) in a test of prediction models with different horizontal resolution. The reanalysis was applied to the height and wind (u and v) fields at standard pressure levels of 1000, 850, 700, 500, 400, 300, and 250 mb.

The operational Hough height and wind fields, their balanced counterparts, and their difference fields at 500 mb are presented first. Although the reanalysis was applied to the entire Northern Hemisphere, maps of only the western part of the Hemisphere covering North American and adjacent oceans will be shown in order to save space and to highlight our area of interest.

The 500-mb height field from the archived Hough analysis and the same field after balanced reanalysis are shown in Figures 1 and 2, respectively. The difference field between the two analyses (Balanced

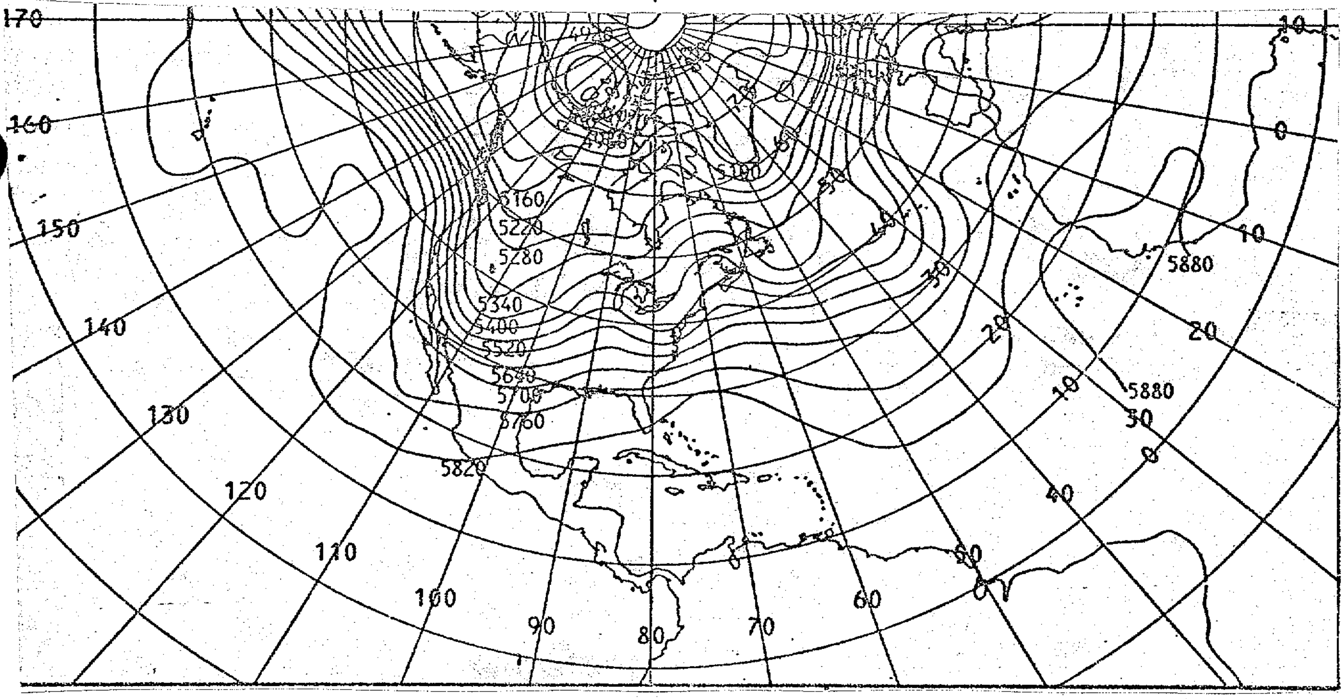

Figure 1. 500-mb Height Field, Hough Analysis, 12Z 9 January 1975 (contour interval: 60 m).

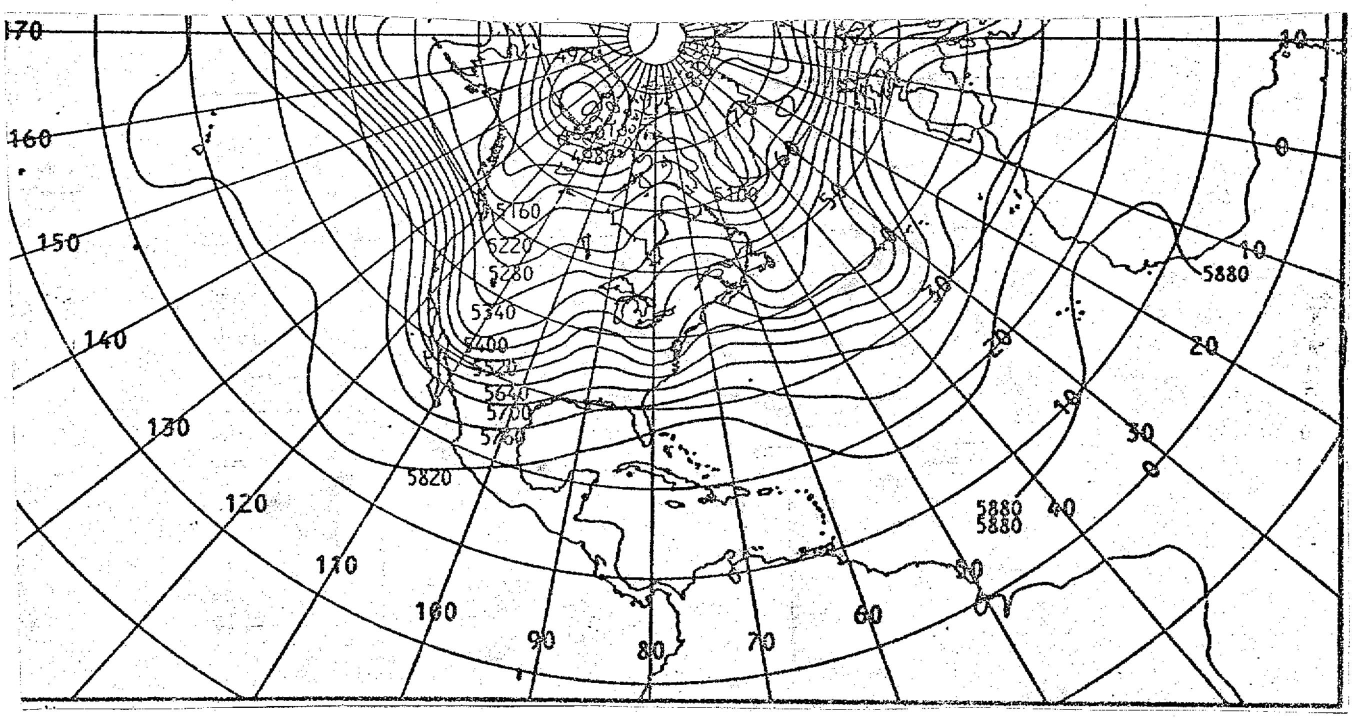

Figure 2. 500-mb Height field, Balanced Analysis, 12Z 9 January 1975 (contour interval: 60 m).

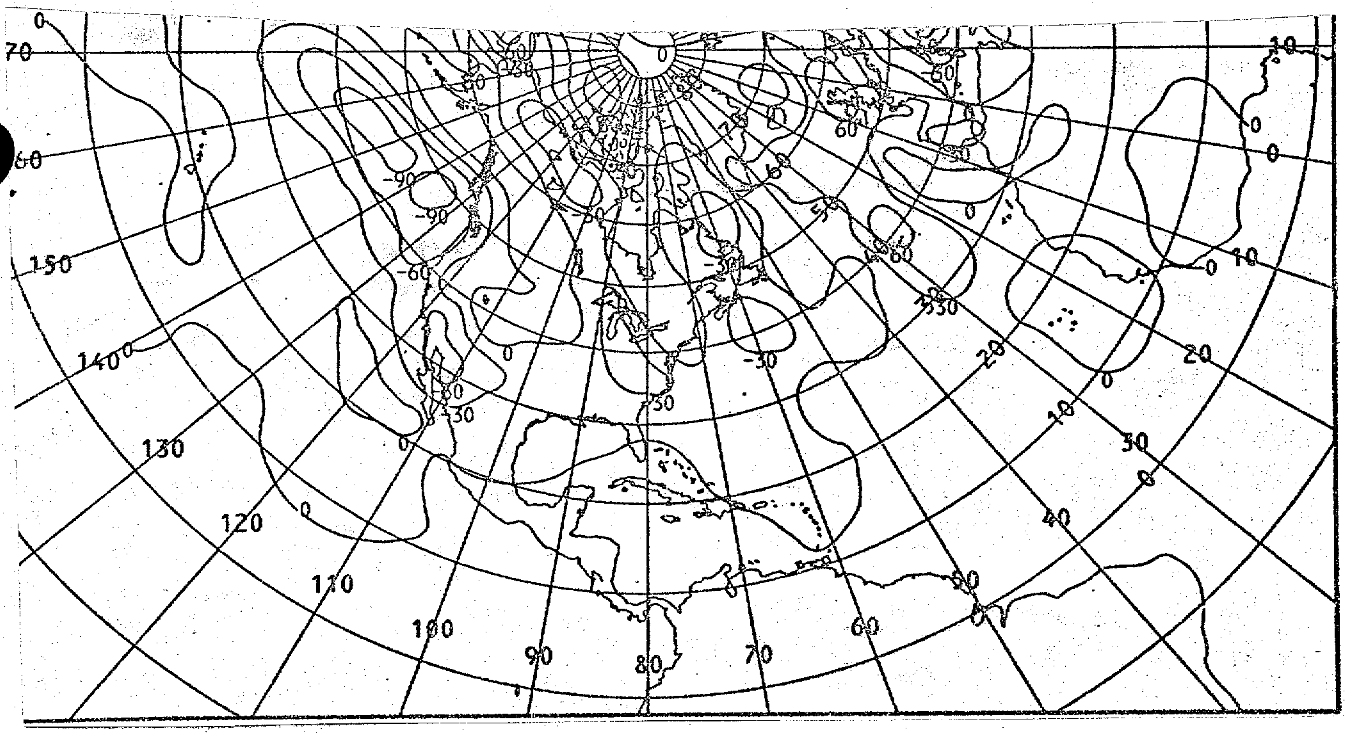

Figure 3. 500-mb Height Difference Field, Balanced Analysis - Hough Analysis, 12Z 9 January 1975 (Contour interval: 30 m).

analysis - Hough analysis) is presented in Figure 3, which shows that height changes over most U.S. and the adjacent Western Atlantic Ocean are small, indicating only modest degree of imbalance in the Hough field. In the Western Atlantic, along longitude 60°W and to the South and North of Gulf of St. Lawrence, two areas of decreasing heights of 30 m or more are noted. The effect of these height changes is to shift the trough along longitude 50°W and to the east of New Foundland slightly westward. The area of decreasing height of 30 m or more in the Eastern U.S., extending from the Upper Great Lakes area southeastward through the Carolinas, had the effect of deepening the existing short wave trough. The area of biggest height change, however, extends from the western U.S. through the eastern part of the North Pacific Ocean, generally coinciding with the area of high degree of imbalance in the Hough field. The maximum height changes over the Pacific, for instance, are more than 90 m, which are located approximately between longitudes 130°W and 160°W.

The largest height change over the western U.S. is an area of 60 m or more, over California, Arizona, and part of Gulf of California.

The wind speed distribution at 500 mb are computed from the distribution of u and v components of the wind. The Hough version of the wind speed field is shown in Figure 4, while the same field for the balanced analysis is in Figure 5. The most prominent area of difference is that of northwesterly jet maximum, which is partially over California coast and partially over the Pacific Ocean. As the result of reanalysis, the speed of the jet maximum was increased from over 40 m sec^{-1} to 50 m sec^{-1} or greater. Over other parts of the U.S. and the Atlantic Ocean, changes in the field of wind speed range from small to insignificant.

The jet stream area, extending from the California coast westward into the eastern part of the North Pacific at 500 mb, persisted at 400, 300, and 250 mb as well. At each level, increased wind speed in the jet maximum is resulted from balanced reanalysis. For example, at 300 mb the reanalysis increases the wind speed in the jet maximum from 70 m sec^{-1} or more to greater than 90 m sec^{-1}. Also at upper levels from 400 to 250 mb, the height fields show decreases in the same general area where jet stream is prevalent, just as they do at 500 mb. Maps of height fields, wind speed fields, and their changes due to reanalysis at upper levels from 400 to 250 mb are not shown here, lest this paper will become too voluminous.

The 850 mb height field from the operational Hough analysis, the same field from the balanced analysis, and the difference field between the two analyses (Balanced analysis - Hough analysis) are presented in Figures 6, 7, and 8, respectively. In the U.S. an area of increasing height (30 m or more) covering the Northern and Central Plain States is

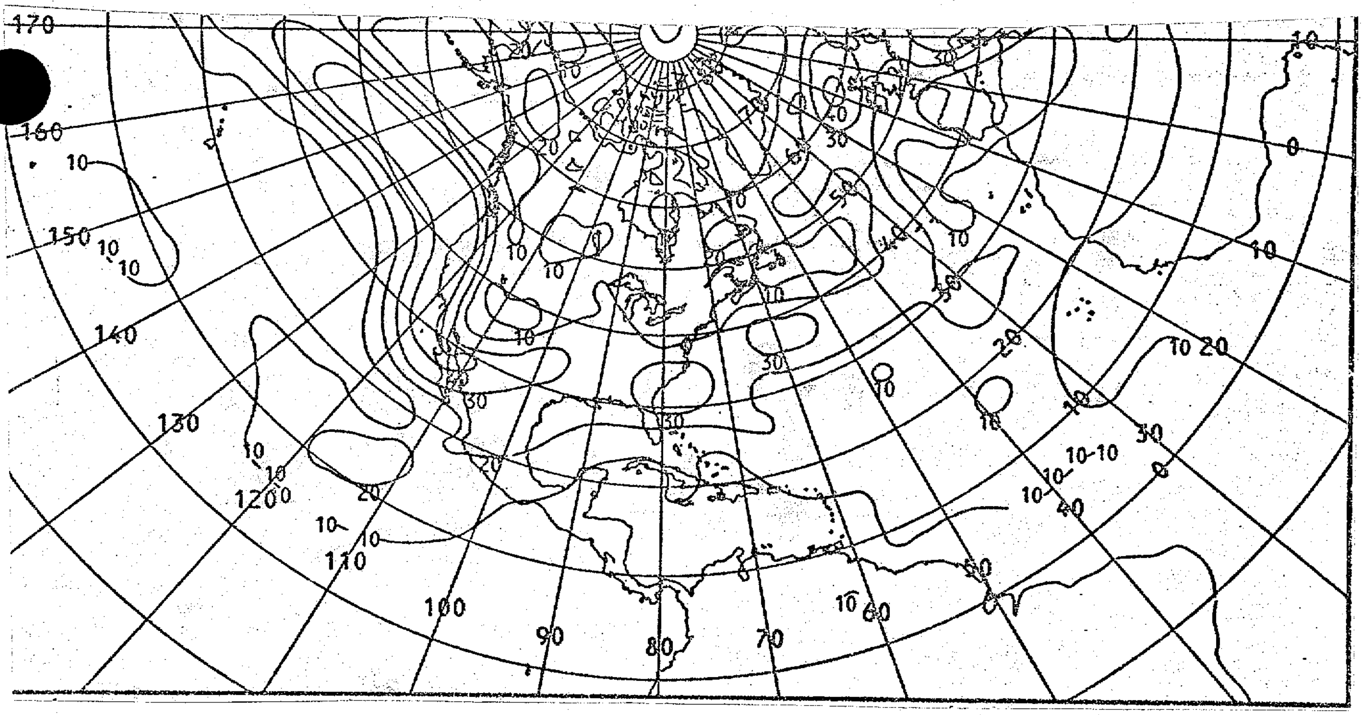

Figure 4. 500-mb Field of Wind Speed, Hough Analysis, 12Z 9 January 1975 (Contour interval: 10 m sec^{-1}).

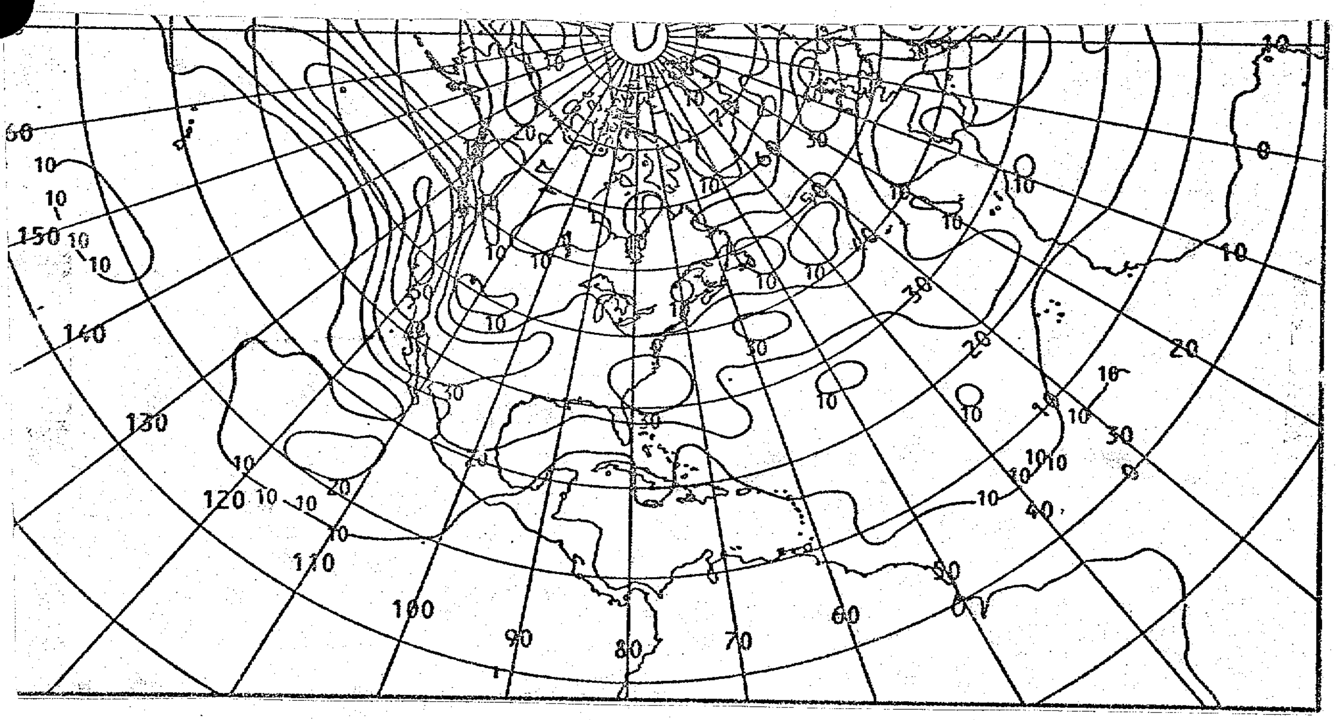

Figure 5. 500-mb Field of Wind Speed, Balanced Analysis, 12Z 9 January 1975 (Contour interval: 10 m sec^{-1}).

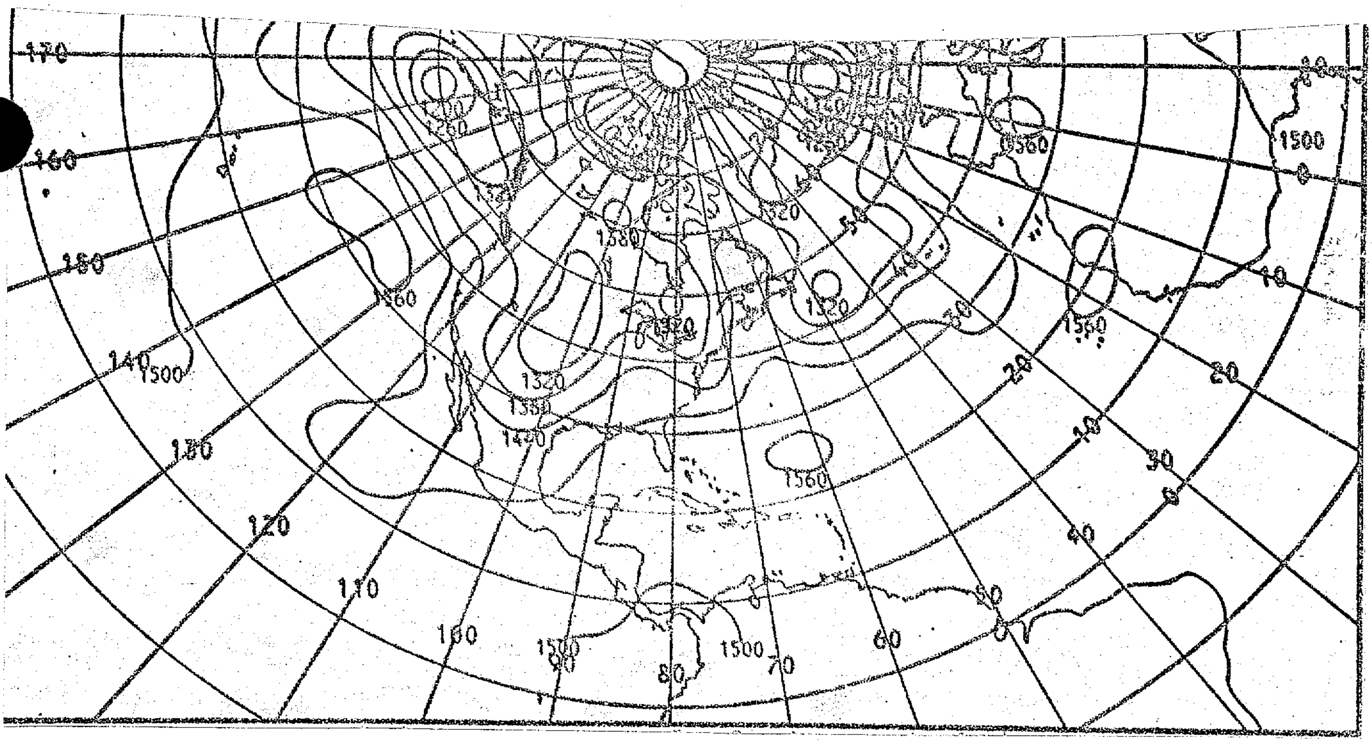

Figure 6. 850-mb Height Field, Hough Analysis, 12Z 9 January 1975 (Contour interval: 60 m).

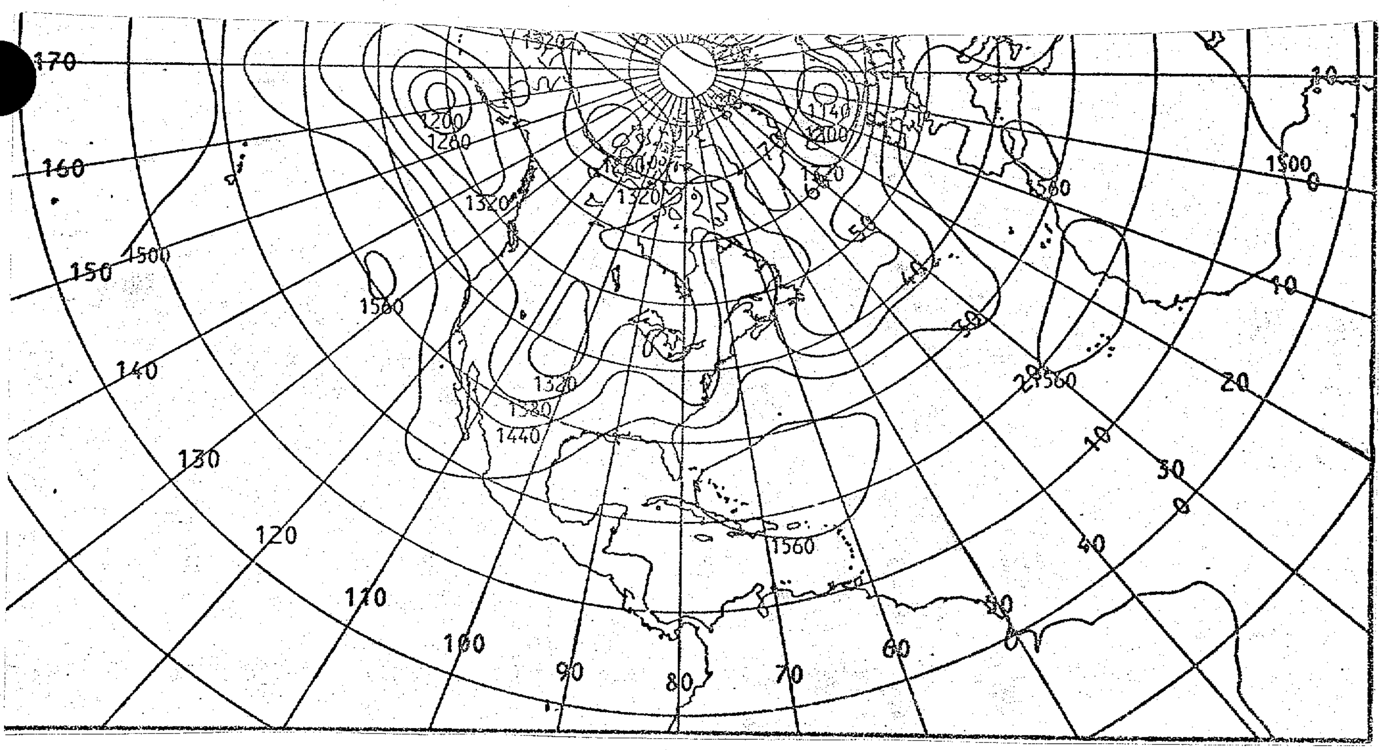

Figure 7. 850-mb Height Field, Balanced Analysis, 12Z 9 January 1975 (Contour interval: 60 m).

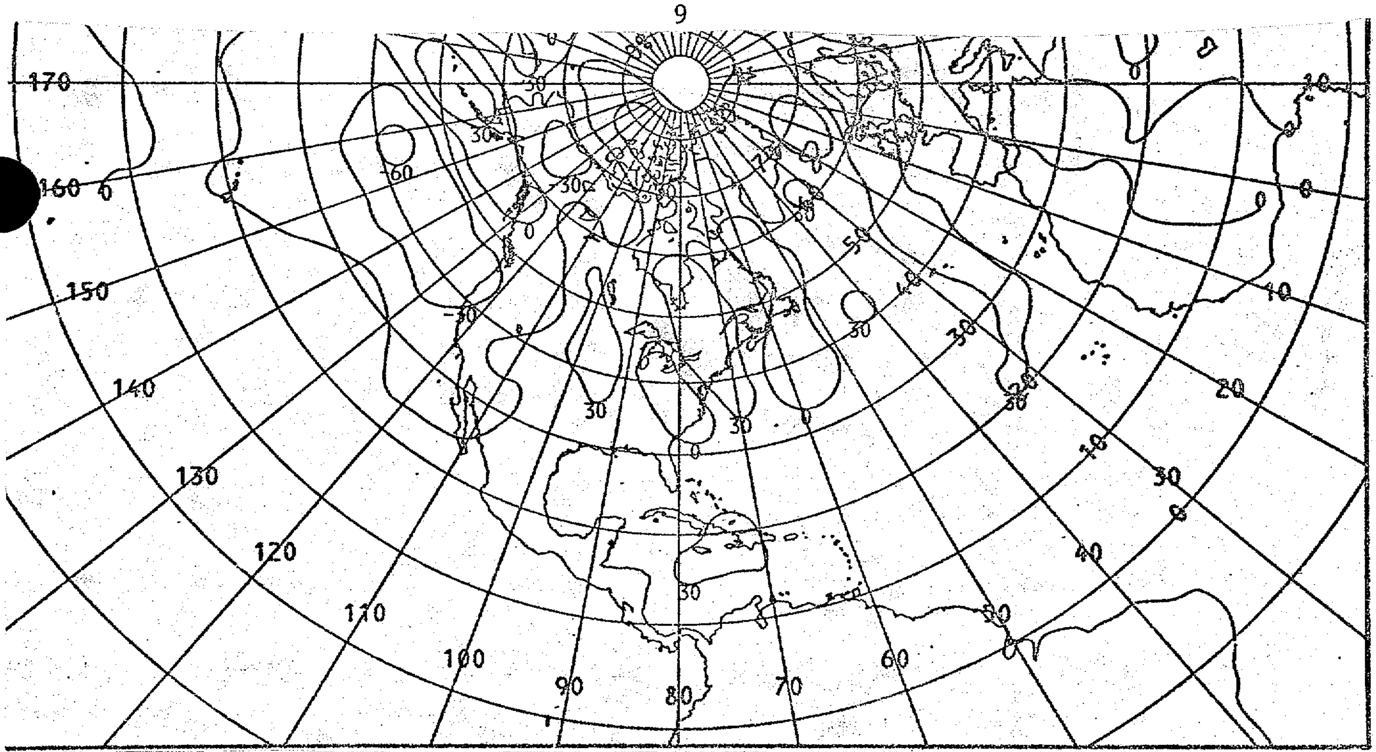

Figure 8. 850-mb Height Difference Field, Balanced Analysis - Hough Analysis, 12Z 9 January 1975 (Contour interval: 30 m).

noted. This height change effectively reduces the size of the area enclosed by the 1320 m contour of a broad low system over much of the U.S. Another area of height increase of similar magnitude (30 m or more) extends from the northeastern U.S. through parts of Ontario and Quebec Provinces in Canada. This increase in height wipes our the enclosed contour of 1320 m in Ontario, just to the northeast of the Great Lakes, and shifts the two neighboring contours of 1380 m and 1440 m westward.

Off the west coast of the U.S and over the eastern North Pacific Ocean, a broad area of decreasing height is prevalent, just as it is at upper levels from 500 mb to 250 mb. The maximum value of height decrease is greater than 60 m and is located north of 40°N, between 150°W and 160°W. As a result of the decreasing height, the area of high center enclosed by the contour of 1560 m is greatly diminished. Another result of this height decrease is for the low system, which is located to the northwest of the high, to shift southward.

The 850 mb field of wind speed from the operational Hough analysis, and the same field from the balanced analysis are shown in Figures 9 and 10, respectively. The change of wind speed over the U.S. continent is generally small except for two areas: (1) to the south of lower Great Lakes and in the region of Michigan, Ohio, and Indiana, there is an area of decreasing wind speed with a maximum value of 6 m sec^{-1}; (2) another area of lower wind speed with a maximum value of 6 m sec^{-1} is located in Wyoming and Montana, mostly to the east of 110°W. Over the adjacent oceans, decreasing wind speed off the coast of Georgia and the Carolinas removes an area of wind speed greater than 20 m sec^{-1} from the Hough analysis. Over the eastern part of the North Pacific, increasing wind speed are responsible for the formation of an area of 20 m sec^{-1} to the south of Aleutian Islands. There is no significant change elsewhere in the Pacific.

Changes in height and wind fields at 700 mb and 1000 mb are generally small except in an area south of the Aleutian Island in the Pacific Ocean, where significant changes are resulted from balanced reanalysis. This area of change is also noted at 850 mb, indicating persistent imbalance in that area throughout lower levels that has been produced by the operational Hough analysis. Elsewhere, the wind and height fields at lower levels from Hough analyses seem to be generally in balance. Maps of Hough analyses and balanced analyses at 700 mb and 1000 mb are not shown here.

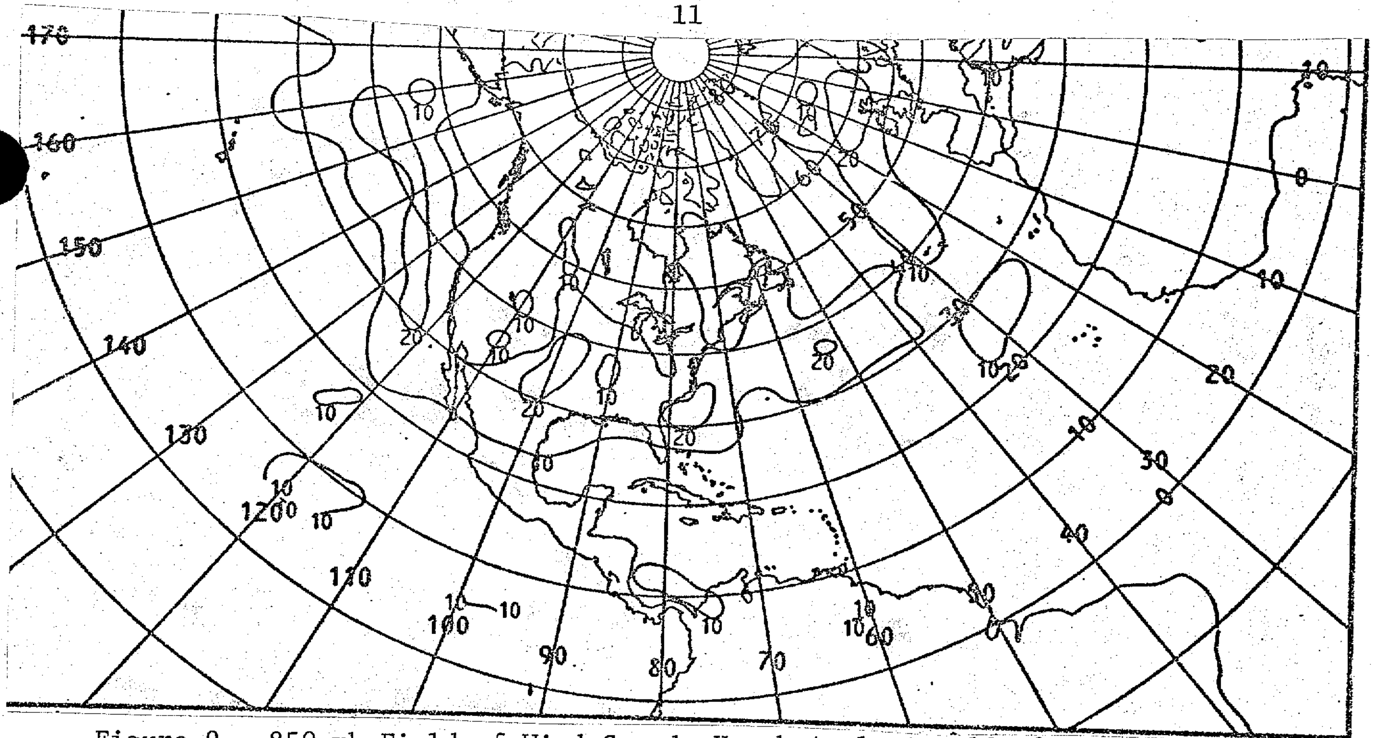

Figure 9. 850-mb Field of Wind Speed, Hough Analysis, 12Z 9 January 1975 (Contour interval: 10 m sec^{-1}).

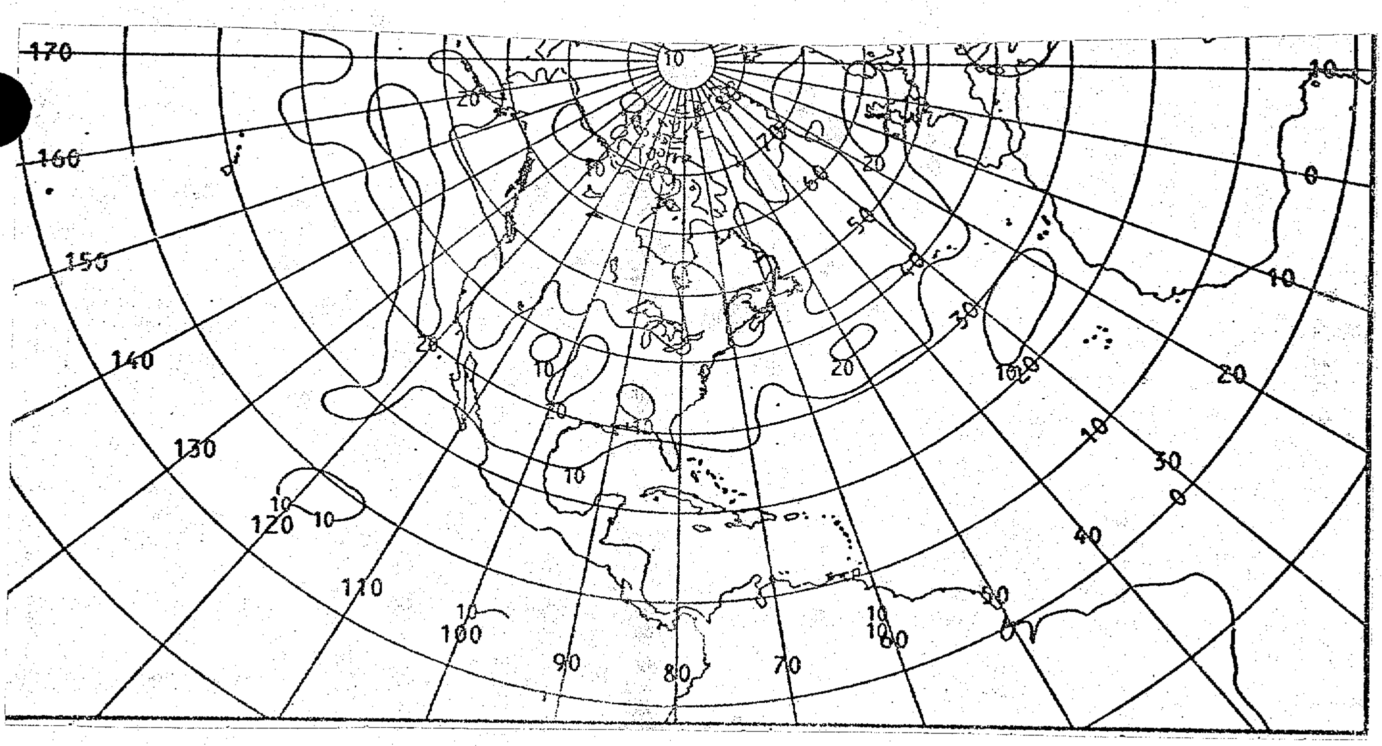

Figure 10. 850-mb Field of Wind Speed, Balanced Analysis, 12Z 9 January 1975 (Contour interval: 10 m sec^{-1}).

4. 7-layer PE Model Forecast With Modified Hough Analyses (Balanced Analyses)

The modified fields of height, u, and v components of the wind from balanced analyses at standard pressure levels from 1000 mb to 250 mb were used to replace their counterparts in the Hough analysis package at 12Z, 9 January 1975. The modified Hough analysis package was then used to generate 7-level PE model forecasts for up to 48 hours.

As the 7-level PE model forecast codes are presently constructed, the initial Hough analyses are subject to an initialization step first before they are used as initial fields for the model forecast. Briefly, the Hough analyses supply, among many other fields, fields of height, u and v components of the wind at standard pressure levels. During the initialization step, the height and wind fields are vertically interpolated from pressure surfaces to sigma surfaces, which are the vertical coordinate surfaces for the forecast model. The interpolated u and v fields are further used to derive a field of nondivergent wind. The sum of this nondivergent wind and the part of irrotational wind, which is extracted from the previous 12-hour forecast of the wind field verifying at the initialization time, becomes the initialized wind field for the forecast model. The modified Hough analysis package, which has the balanced height and wind fields from 1000 mb to 250 mb, is subject to the same initialization process. Whether the initialization procedure affects the balanced state in the height and wind fields at the above-mentioned pressure levels is a matter of conjecture at this time.

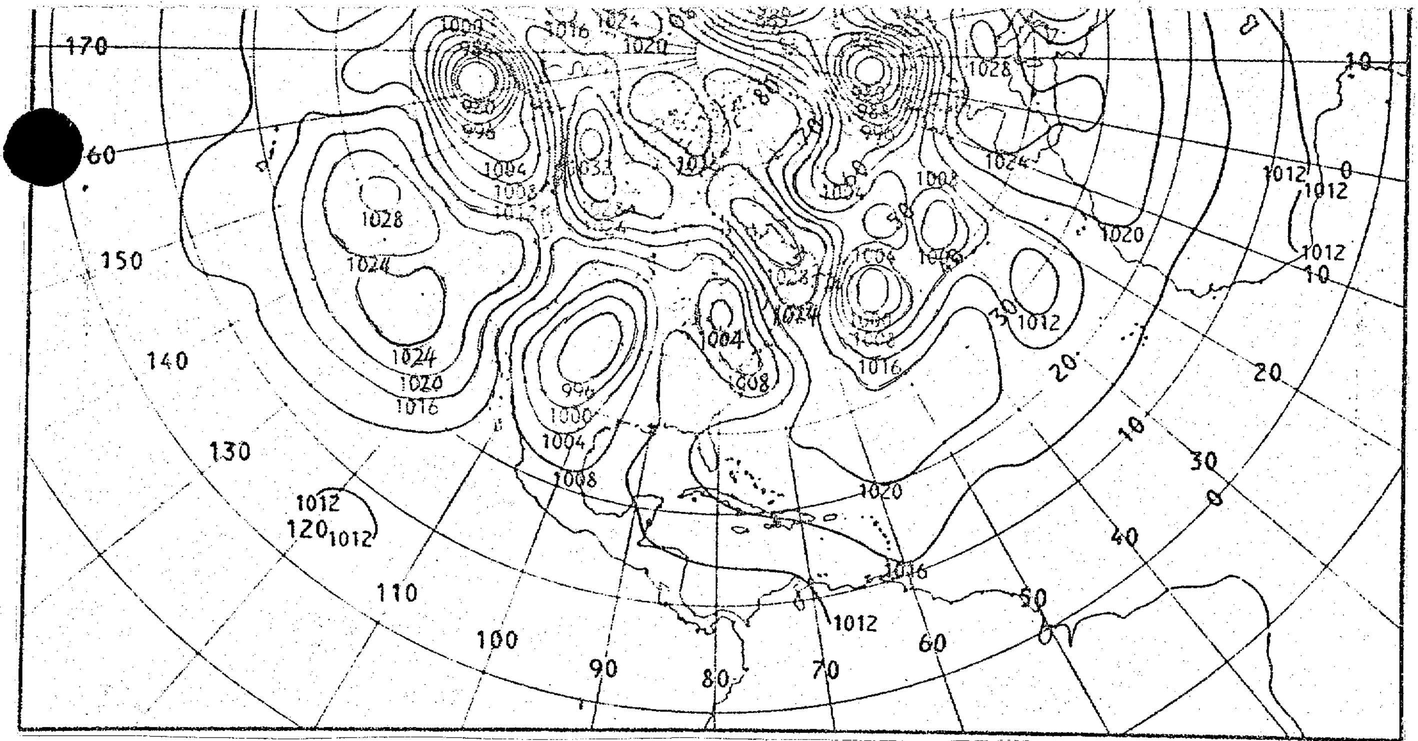

Figure 11. Sea-level Pressure Field, Hough Analysis, FOO, 12Z 9 January 1975 (Contour interval: 4 mb).

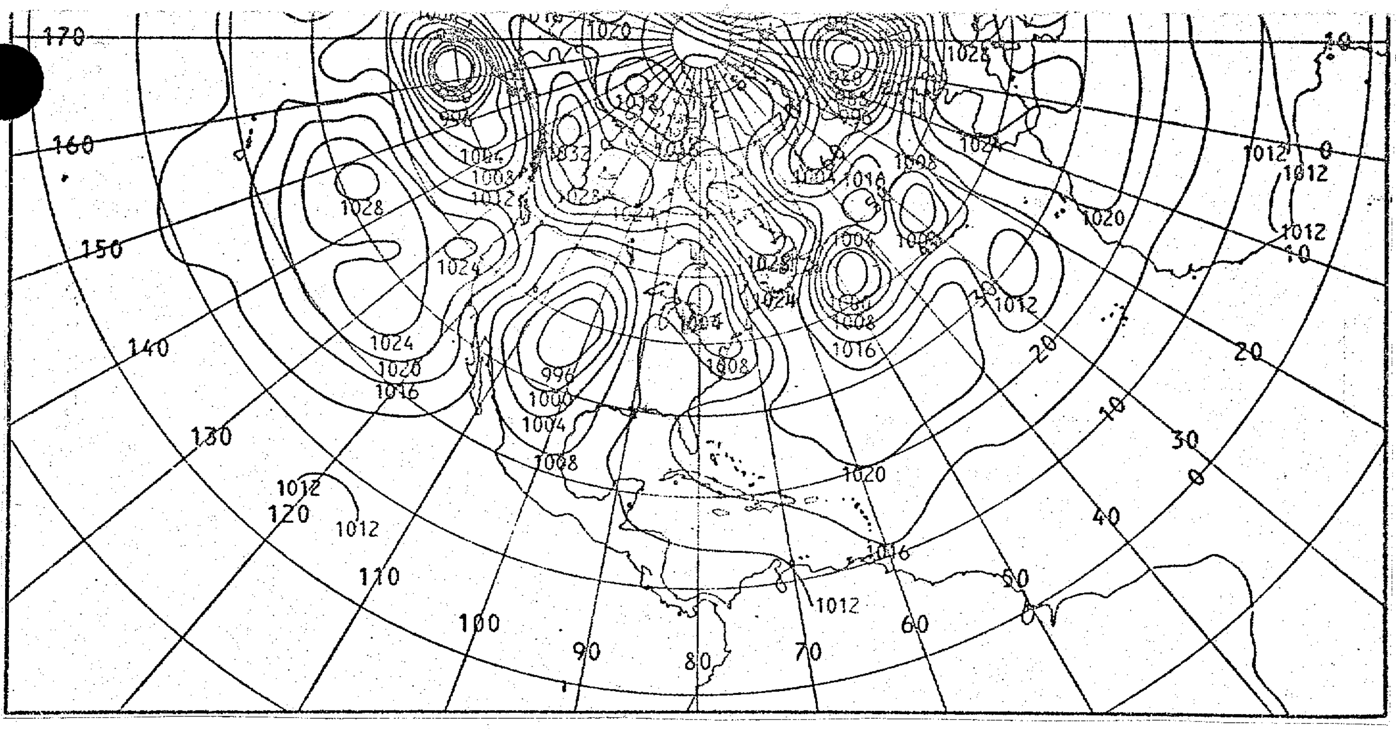

Figure 12. Sea-level Pressure Field, Balanced Analysis, FOO, 12Z 9 January 1975 (Contour interval: 4 mb).

4.1 The Initialized State at 12Z, 9 January 1975

As was previously mentioned, the case of 12Z 9 January 1975 is well known at NMC as a typical "locked-in-error" case, and was used by Phillips (1978) in a test of prediction models with different horizontal resolution. Phillips reported that both the LFM and 6-layer PE models produces a large error in forecast, and that the fine-mesh 7-layer PE model (mesh size 174 km at 45^o latitude versus 349 km at same latitude for 6-layer PE model) had shown some improvement over the 6-layer PE model. In our experiment, fine-mesh 7-layer PE model forecast up to 48 hours was made with two different initial analyses: (a) operational Hough analysis without modification, (b) modified Hough analysis with balanced height and wind fields at standard pressure levels from 1000 mb to 250 mb.

The sea-level pressure field from Hough analysis and that from balanced analysis are shown in Figures 11 and 12, respectively. There is little difference between the two pressure patterns over the U.S., where a huge low pressure system covered the western half of the country with a central pressure of 996 mb. Another low of 1004 mb was located in the eastern U.S. During the first 12 hours, the low of 996 mb filled slightly and moved from Colorado-Kansas border southward to western Texas. For the next 36 hours, this low system deepened and its center moved northeastward through Oklahoma, Iowa to Lake Superior with a central pressure of 970 mb. We shall focus our attention on this low system in our model forecast comparison at sea level.

The 500 mb height field from the archived Hough analysis and that from balanced analysis were shown previously in Figures 1 and 2, respectively. In 24 hours, the deep trough system over Utah, Nevada, Idaho in the western U.S. would move eastward to locate in Texas, Kansas, and Nebraska, with the associated center of vorticity maximum moving from Arizona to the Texas Panhandle and increasing slightly in intensity (from 18 to 20 units). During the next 24 hours, the system would move northeastward in a rapid and accelerated pace. At 12Z, 11 January 1975, the end of the 48-hour period, a low center of 5069 m would be located over Green Bay, Wisconsin. More details about the synoptic situation can be found in Phillips (1978).

4.2 The 48-hour Forecast

The 48-hour forecast 500-mb height field from 7-layer model with balanced analysis is shown in Figure 14, while that from 7-layer model with Hough analysis is in Figure 15. The forecast charts should be compared with the analyzed chart at verifying time (12Z, 11 January 1975) in Figure 13. By comparison the forecast low system over the U.S. from 7-layer model with balanced analysis is better than that from the same model with unmodified Hough analysis. The model forecast with balanced analysis had a value of 5146 m for the low center, locating over Iowa-South Dakota border, while the forecast with Hough analysis had 5166 m for the low center, that was forecast to be over Iowa-Kansas border. The verifying analysis shows the low center to be in Green Bay, Wisconsin, and to have a central value of 5069 m. The central values and locations of the low centers in the charts were obtained from grid-print maps, based upon which the charts in Figures 13 to 15 were produced on microfilms. It is evident that both forecasts are slow on the movement of the low system, and that the forecast with balanced analysis is faster than that with Hough analysis.

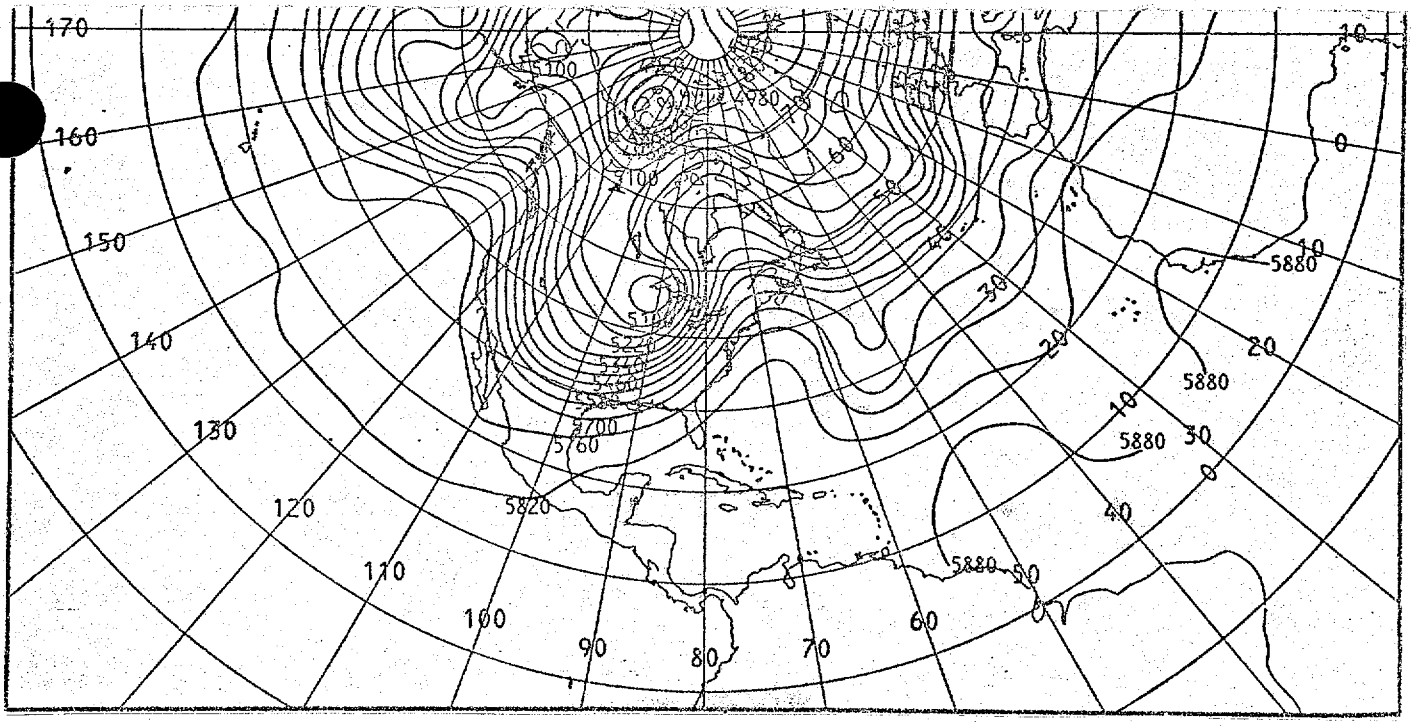

Figure 13. 500-mb Height Field, Hough Analysis, 12Z 11 January 1975 (Contour interval: 60 m).

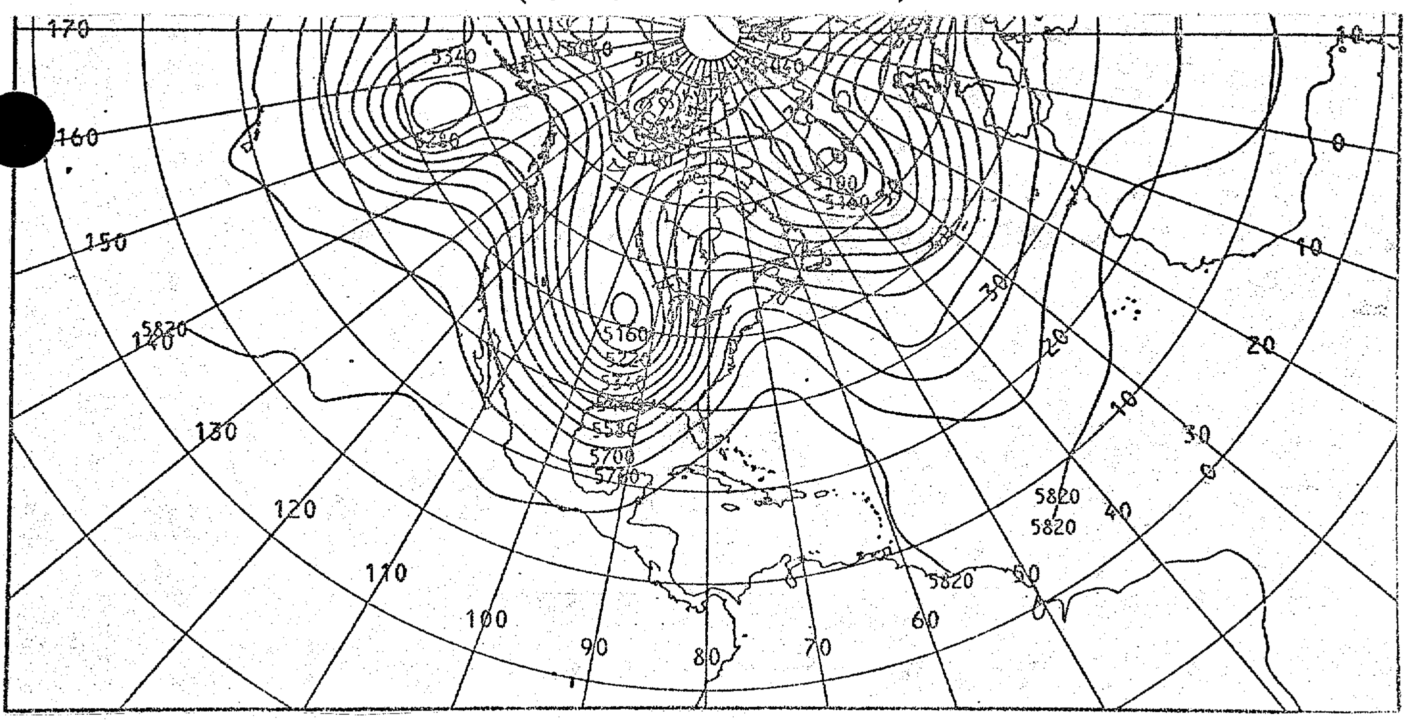

Figure 14. 48-hour Forecast 500-mb Height Field, 7-layer Model With Balanced Analysis, Valid at 12Z 11 January 1975 (Contour inteval: 60 m).

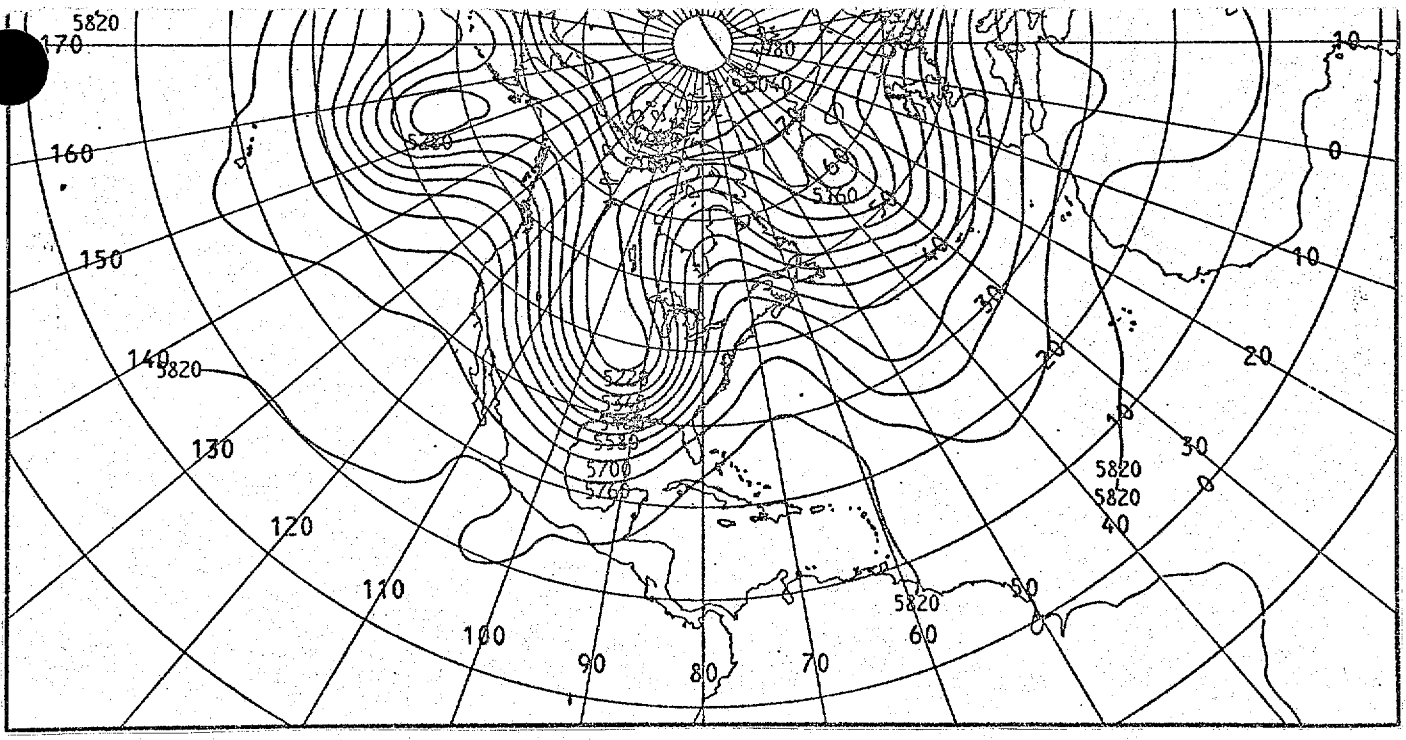

Figure 15. 48-hour Forecast 500-mb Height Field, 7-layer Model With Hough Analysis, Valid at 12Z 11 January 1975 (Contour interval: 60 m).

The 500-mb height error field of 48-hour forecast with balanced analysis, and that with Hough analysis are shown in Figures 16 and 17, respectively. The error patterns over the U.S. are similar with the following notable features: (1) the negative error over the Gulf states has the smaller maximum value of 120 m for the forecast with balanced analysis, compared with 150 m for that with Hough analysis. (2) Over Green Bay, Wisconsin, where the low center was observed and maximum positive error was found, the error of 240 m is the same for both forecasts. (3) Over the western U.S. the positive error has generally been reduced. Phillips (1978) reported a 48-hour forecast error near Salt Lake City for all model forecasts, that ranges from 120 m for 7-layer PE models, to 240 m for NGM models. The forecast with balanced analysis yields a forecast error near Salt Lake City of only 71 m.

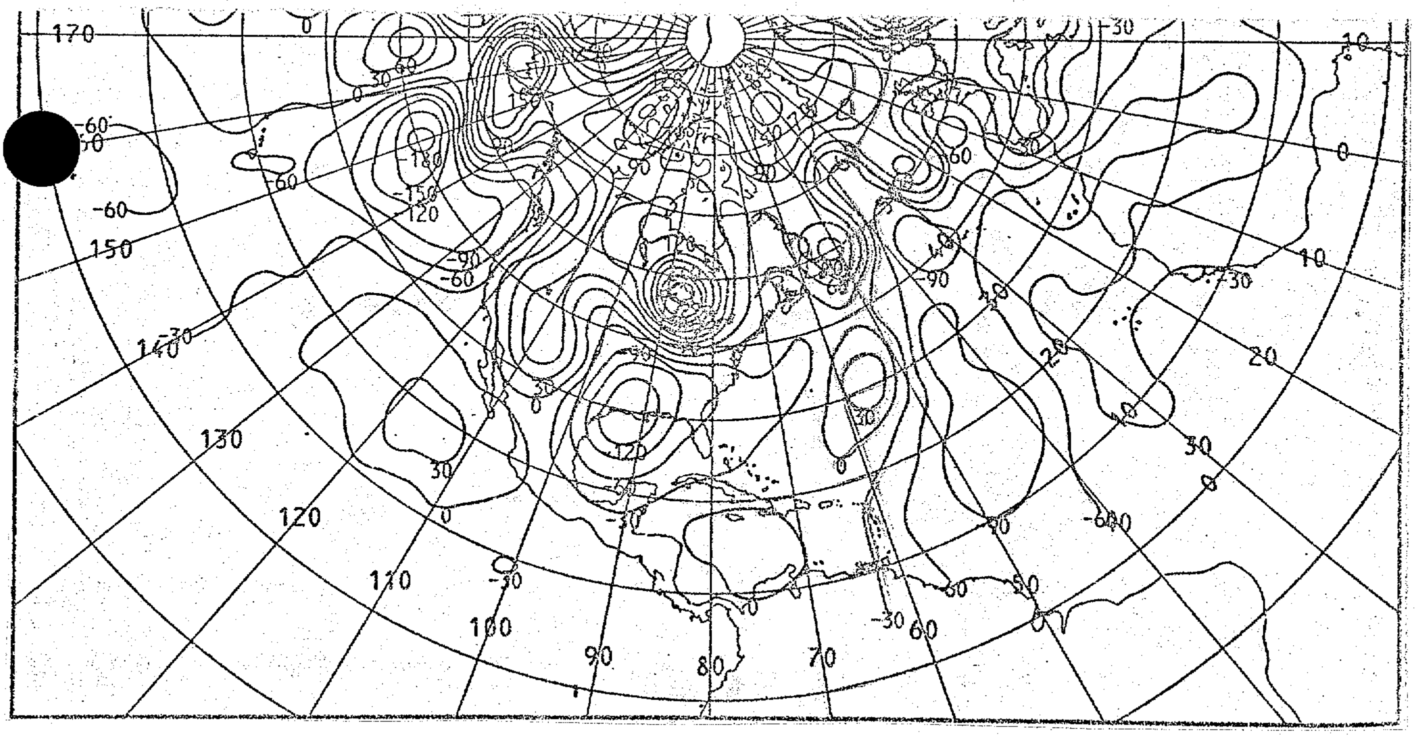

Figure 16. Errors of 48-hour Forecast 500-mb Height Field, Forecast With Balanced Analysis - Verifying Analysis at 12Z 11 January 1975 (Contour interval: 30 m).

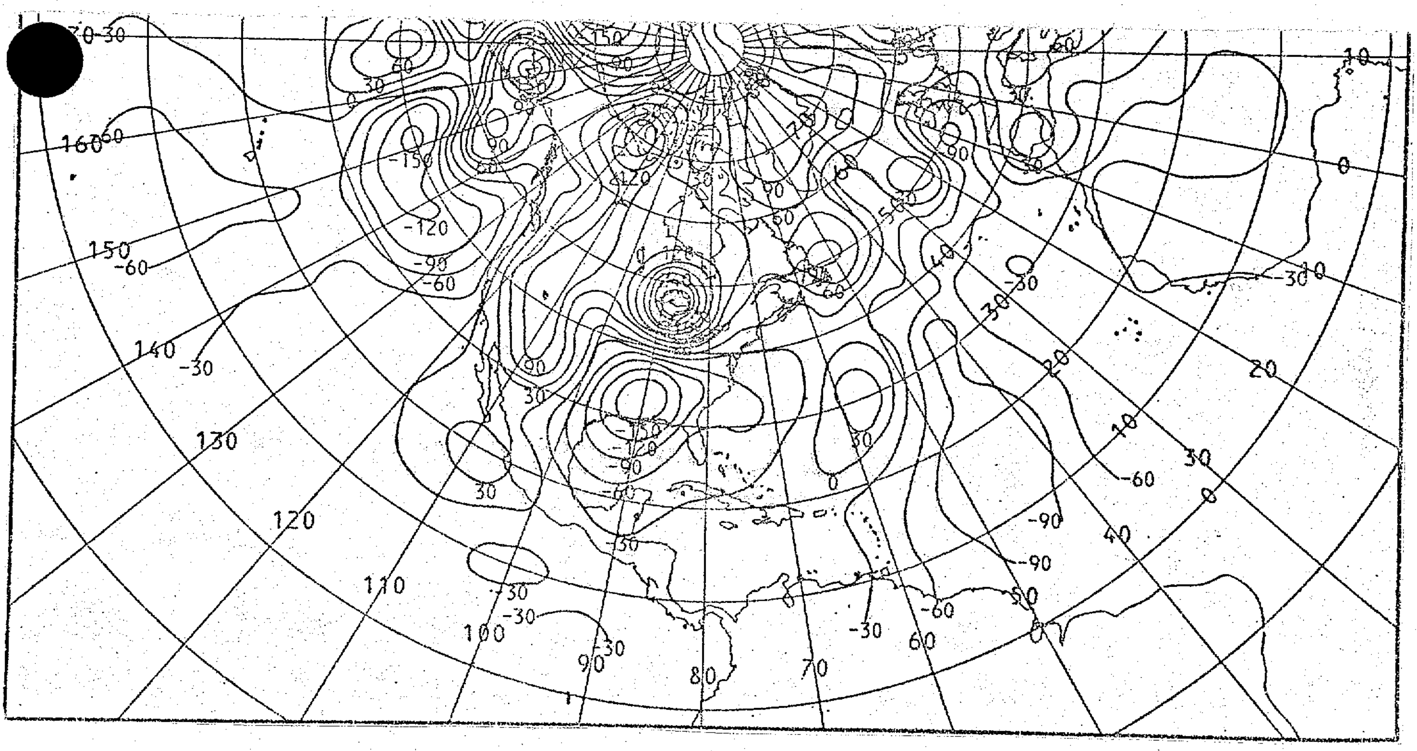

Figure 17. Errors of 48-hour Forecast 500-mb Height Field, Forecast With Hough Analysis - Verifying Analysis at 12Z 11 January 1975 (Contour interval: 30 m).

The verifying sea-level pressure field, 48-hour forecast with balanced analysis, and the same forecast with Hough analysis are shown in Figures 18, 19, and 20, respectively. Against the verifying low center of 970 mb over Lake Superior, both forecasts were in error: forecast with balanced analysis had a low 977 mb just to the south of Duluth, Minnesota; that with Hough analysis had a low of 977 mb in Green Bay, Wisconson. Again, the central pressures were obtained from grid-print maps, and are not displayed in Figures 18 to 20. Neither forecast had isobars around the low center as tight as those observed. Both forecasts had cyclonic circulation extended too far south, although forecast with balanced analysis was better in extending it only to Arkansas, well short of Gulf of Mexico.

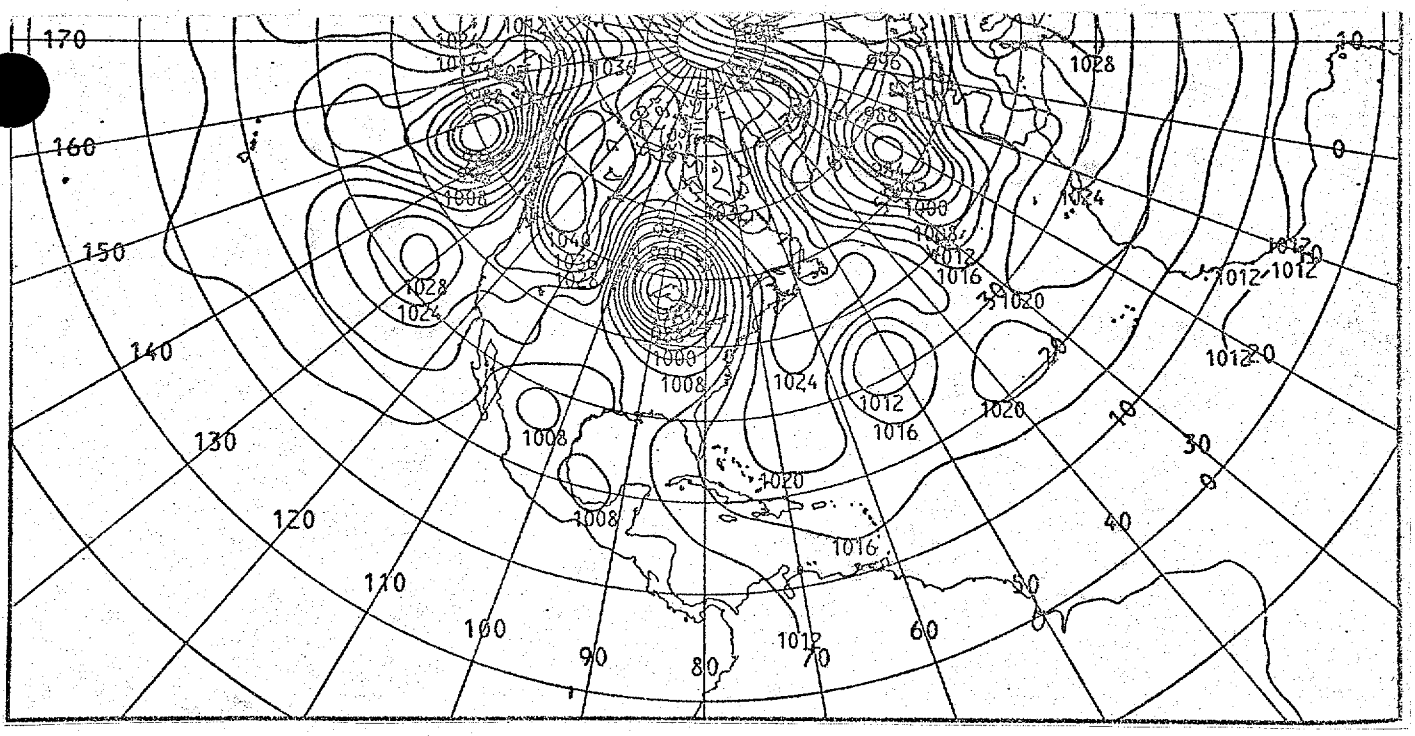

Figure 18. Sea-level Pressure Field, Hough Analysis, 12Z 11 January 1975
(Contour interval: 4 mb).

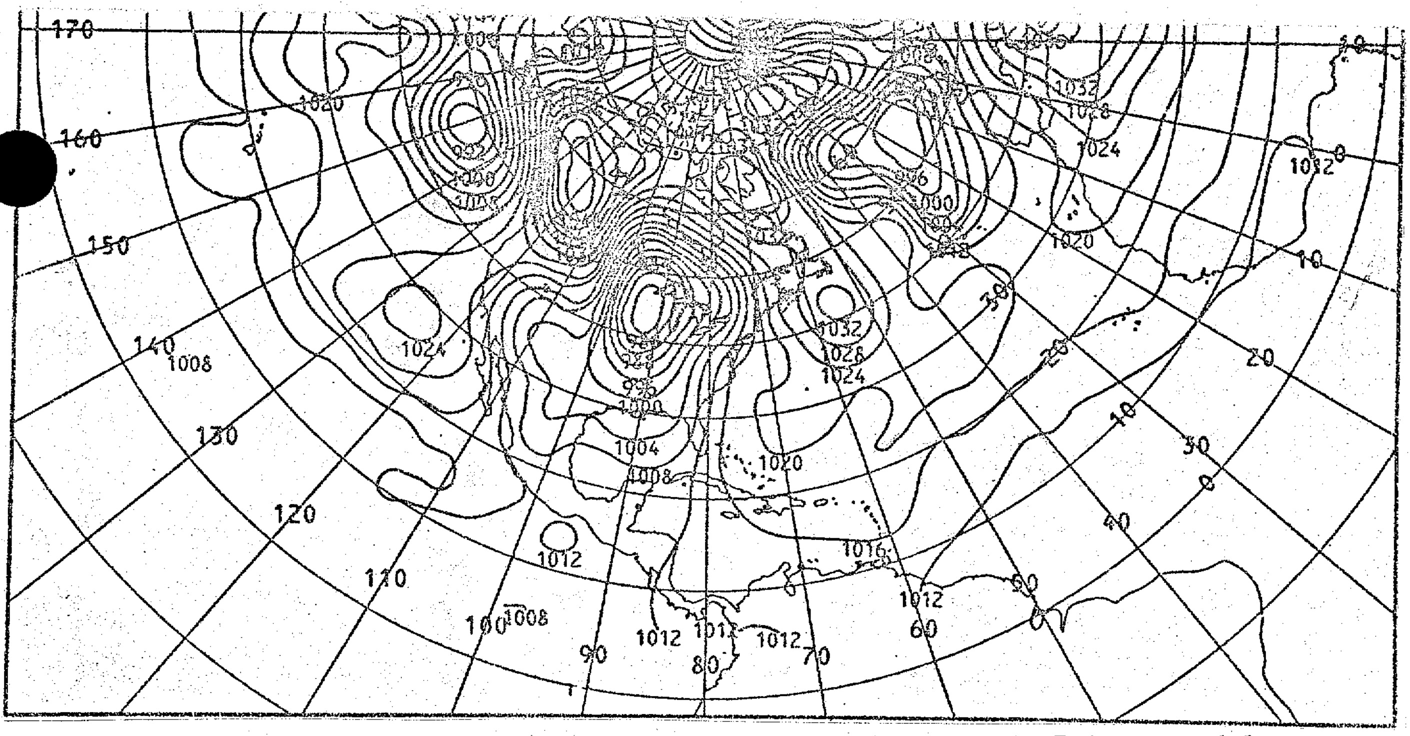

Figure 19. 48-hour Forecast Sea-level Pressure Field, 7-layer Model With Balanced Analysis, Valid at 12Z 11 January 1975 (Contour interval: 4 mb).

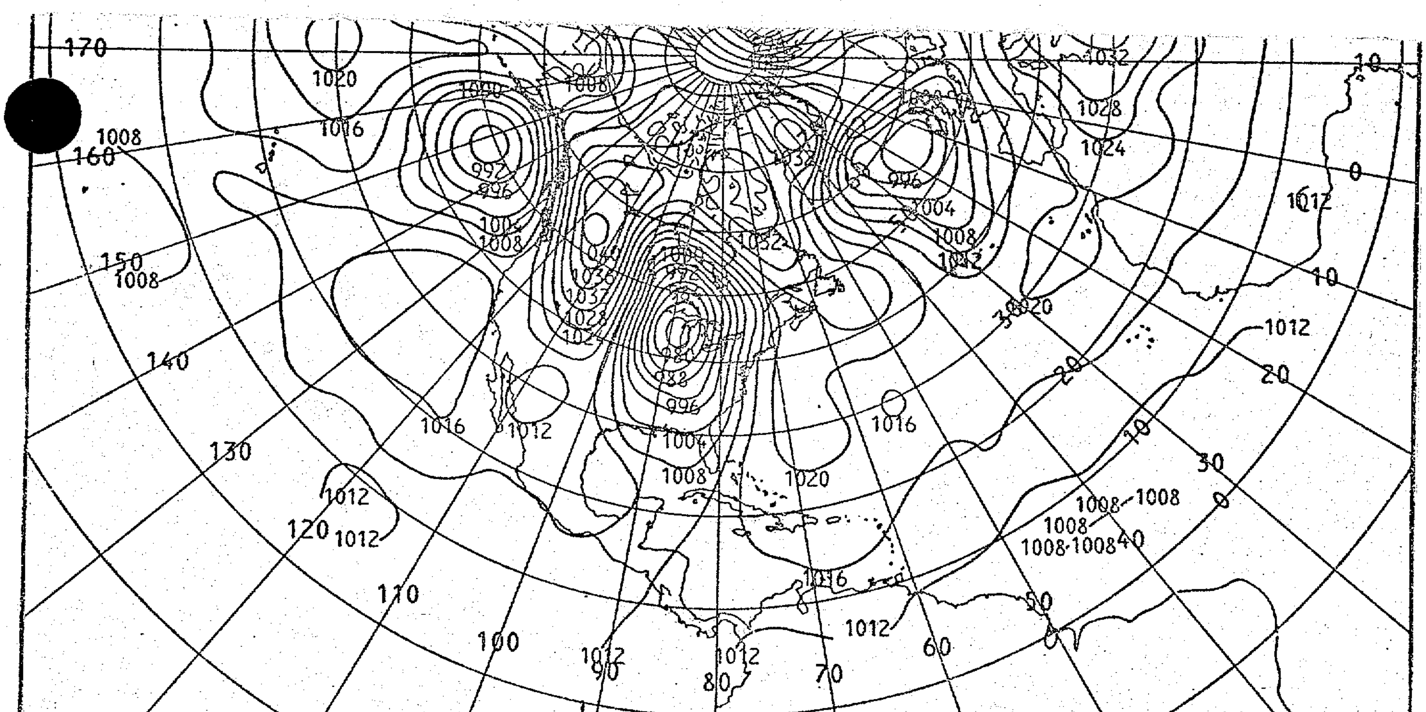

Figure 20. 48-hour Forecast Sea-level Pressure Field, 7-layer Model With Hough Analysis, Valid at 12Z 11 January 1975 (Contour interval: 4 mb).

Verification statistics were computed with the aid of computer codes developed by the System Evaluation Branch, Development Division, NMC. Those for the area (25°-60°N, 35°-130°W) covered in our display maps at 850 mb, 500 mb, and 250 mb, and verifying at 24 and 48 hours are tabulated in Table 1. It is indicated from Table 1: (1) at 48 hours forecast height fields with balanced analyses are statistically more accurate than those with operational Hough analyses. It is speculated that increases in wind speed in the jet maxima at upper levels off the western coast of the U.S. might have accelerated the movement of the low system in the western U.S., thus helping the forecast height fields with balanced analyses. However, mixed results are shown in temperature and wind verification statistics. In fact, in most categories forecasts with balanced analyses are less accurate than their counterparts with Hough analyses. (2) At 24 hours, height forecasts with balanced analyses are slightly better than those with Hough analyses at 850 mb and 250 mb, but worse at 500 mb. Again, mixed results are shown in temperature and wind forecasts. Those with balanced analyses are mostly worse than those with Hough analyses.

5. Summary

The primary objective of this paper is to show the feasibility of applying an iterative variational method with balanced constraint to hemispheric analysis and prediction. Procedures of reanalyzing height and wind fields from operational Hough analyses, and of incorporating the reanalyzed fields to generate model prediction have been demonstrated with an archived case. It is not the intention of this paper to use a one-case experiment to claim any advantage or disadvantage of this variational method. It is clear, however, that balanced analyses did not produce forecasts that were worse than those with Hough analysis in this case.

It should be pointed out that geostrophic balance to some degree is contained in Hough analyses. Thus, imbalance between the Hough height and wind fields, if any, should generally not be excessive. The logical extension, therefore, is to apply the balanced variational method to the LFM and Optimum Interpolation (OI) analyses, in which there is no built-in balance between the height and wind fields.

6. References

Gerrity, J. P. and R. Chu, "An iterative variational method for adjusting isobaric wind and geopotential to satisfy the balance equation," Office Note 192, NMC, NOAA, October 1978.

Phillips, N. A., "A test of finer resolution," Office Note 171, NMC, NOAA, February 1978.

Table 1

Forecasts from 12Z 9 Jan. 1975
Verified Against Radiosondes 25° - 60°N, 35° - 130°W

(850 mb)

		S1		Mean Errors		RMS Errors	
		Hough	Balanced	Hough	Balanced	Hough	Balanced
48 hr.	Height (m)	41.6	41.5	-13.2	- 8.0	50.7	49.1
	Temperature (°C)	49.9	53.9	1.7	2.4	4.9	5.3
	RMS Vector Wind ($m\ sec^{-1}$)			9.9	10.2	11.2	11.5
24 hr.	Height (m)	45.3	44.1	12.8	11.8	35.2	23.3
	Temperature (°C)	62.4	63.2	0.7	0.5	3.5	4.3
	RMS Vector Wind ($m\ sec^{-1}$)			7.7	8.5	8.9	9.8

(500 mb)

		S1		Mean Errors		RMS Errors	
		Hough	Balanced	Hough	Balanced	Hough	Balanced
[illegible]r.	Height (m)	39.6	37.0	0.9	- 1.1	89.7	79.1
	Temperature (°C)	50.9	53.2	0.6	- 0.4	4.0	3.9
	RMS Vector Wind ($m\ sec^{-1}$)			14.8	14.1	16.6	15.8
24 hr.	Height (m)	31.9	32.7	2.6	-14.5	44.7	37.0
	Temperature (°C)	57.2	59.4	- 0.3	- 1.7	2.3	2.9
	RMS Vector Wind ($m\ sec^{-1}$)			8.7	8.9	10.3	10.7

(250 mb)

		S1		Mean Errors		RMS Errors	
		Hough	Balanced	Hough	Balanced	Hough	Balanced
48 hr.	Height (m)	40.8	37.9	-14.8	-14.6	132.3	117.4
	Temperature (°C)	90.4	97.4	- 0.2	3.9	4.4	5.7
	RMS Vector Wind ($m\ sec^{-1}$)			20.0	19.1	22.0	21.8
24 hr.	Height (m)	27.7	27.2	- 1.8	-23.2	53.6	46.0
	Temperature (°C)	78.2	63.2	1.2	5.0	3.0	5.7
	RMS Vector Wind ($m\ sec^{-1}$)			10.5	12.7	11.8	13.7